Absence of trapped modes for a Y-shaped junction of open waveguides

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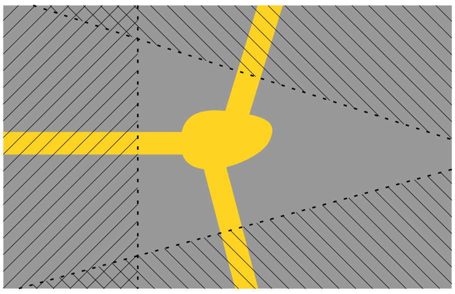
(a common work with Anne-Sophie Bonnet-Ben Dhia and Sonia Fliss)

We consider the two-dimensional Helmholtz equation

$$\Delta u + k^2 n^2\, u = 0 \quad \text{in } \mathbb{R}^2, \tag{1}$$

where $n = n(x)$ is a bounded function with represents the refraction index of a Y-shaped junction of open waveguides, as shown below.



The purpose of this talk is to prove that if the angles between the three branches of waveguides are all greater than or equal to $\pi/2$, then the only solution $u \in L^2(\mathbb{R}^2)$ to the above Helmholtz equation is $u \equiv 0$. This shows the absence of so-called *trapped modes* (or *bound states*) of such a structure or equivalently, the absence of eigenvalues embedded in the continuous spectrum of the operator $n^{-2}\Delta$.

The first similar result is presented in [2] for a rectilinear junction of two waveguides. For the case considered here, the proof combines the ideas used in [2, 3] with an original technique recently proposed in [1]. The key point is to use modal representations of u in the three hatched half planes represented above. Matching these representations yields the result thanks to an analyticity argument.

References

[1] A.-S. Bonnet-BenDhia, S. Fliss, C. Hazard and A. Tonnoir, A Rellich type theorem for the Helmholtz equation in a conical domain, C. R. Acad. Sci. Paris, Ser. I, 354 (2016) 27–32.

[2] A.-S. Bonnet-Ben Dhia, B. Goursaud, C. Hazard, Mathematical analysis of the junction of two acoustic open waveguides, SIAM J. Appl. Math. 71 (2011) 2048–2071.

[3] C. Hazard, On the absence of trapped modes in locally perturbed open waveguides, IMA J. Appl. Math. (2014) doi: 10.1093/imamat/hxu046.