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Spectral analysis of the magnetic Laplacian with vanishing magnetic field

The spectral theory of the Schrödinger operator with magnetic field and semiclassical parameter generates a lot of interest. It is linked to the Ginzburg-Landau functional and applies to the physical study of surface superconductivity. The subject is now well known in dimension two, but most studies concern the case of non vanishing magnetic fields.

This talk will be devoted to the spectral analysis of a self-adjoint realization of the operator :

$$\mathcal{P}_{h,\mathbf{A},\Omega} = (-ih\nabla + \mathbf{A})^2 = \sum_{j=1}^{2} (-ih\partial_{x_j} + A_j)^2,$$

in the semiclassical regime, where the magnetic field $\mathbf{B} = \partial_{x_1} A_2 - \partial_{x_2} A_1$ vanishes along a regular curve (with Ω a bounded and simply connected domain of $\mathbb{R}^2$ with smooth boundary and $\mathbf{A} \in \mathcal{C}^\infty(\overline{\Omega}, \mathbb{R}^2)$).

We will be especially interested in the first asymptotic term of the lowest eigenvalue $\lambda_1(h)$ when the parameter h goes to 0. The talk will be illustrated by numerical simulations based on the Finite Elements Library "Mélina++" (developed at the University of Rennes 1). The numerical difficulties related to the high oscillation of the phase (in the eigenfunctions expression) is circumvented by a polynomial interpolation of high degree.